

## Minimal flavour violation and beyond

---

**Thorsten Feldmann and Thomas Mannel**

*Theoretische Physik 1, Fachbereich Physik, Universität Siegen,  
D-57068 Siegen, Germany*

*E-mail:* [feldmann@hep.physik.uni-siegen.de](mailto:feldmann@hep.physik.uni-siegen.de), [mannel@physik.uni-siegen.de](mailto:mannel@physik.uni-siegen.de)

**ABSTRACT:** Starting from the effective-theory framework for Minimal Flavour Violation, we give a systematic definition of next-to-minimal (quark) flavour violation in terms of a set of spurion fields exhibiting a particular hierarchy with respect to a small (Wolfenstein-like) parameter. A few illustrative examples and their consequences for charged and neutral decays with different quark chiralities are worked out in some detail. Our framework can be used as a model-independent classification scheme for the parameterization of flavour structure from physics beyond the Standard Model.

**KEYWORDS:** Beyond Standard Model, Quark Masses and SM Parameters, Rare Decays .

JHEP02(2007)067

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Quark-flavours in the standard model</b>	<b>2</b>
2.1 The role of custodial SU(2) in flavour physics	3
<b>3. Minimal flavour violating new physics</b>	<b>4</b>
<b>4. Defining non-Minimal Flavour Violation (nMFV)</b>	<b>6</b>
4.1 Example 1: nMFV spurion $Y_R$ and Froggatt-Nielsen power-counting	7
4.2 Example 2: nMFV spurion $Y_R$ with democratic power-counting	11
4.3 Phenomenological implications	12
4.3.1 nMFV: right handed spurion $Y_R$	12
4.3.2 nMFV: octet spurions	14
<b>5. Conclusions</b>	<b>15</b>

---

## 1. Introduction

Despite the enormous progress in the description of elementary particle interactions, the notion of flavour remains a mystery. In the standard model (SM) the flavour structure is parameterized by the Yukawa couplings, which yield the masses and the mixing angles of the CKM matrix as physical parameters. There are, at the moment, no convincing models for the observed masses and mixing pattern, at least not at the quantitative level. The unification of forces in Grand Unified Theories (GUTs, see e.g. [1–3]) mainly concerns the gauge sector, while flavour is still implemented by a triplication of matter multiplets for the different fermion families, and the precise mechanism creating masses and mixings is parameterized in the symmetry breaking sector. In supersymmetric extensions of the SM, the soft SUSY-breaking terms in the Lagrangian even add new sources of flavour structure (see e.g. [4] and references therein for a phenomenological discussion).

Due to the lack of a theory of flavour, we have no clear idea what effects one may expect beyond the parameterization encoded in the SM Yukawa couplings. With the next era of particle colliders in front of us, and the hope to produce and detect new particles and interactions, we also have to improve the theoretical framework to discuss flavour structure beyond the SM. A well-known example is the concept of minimal flavour violation (MFV [5], for an earlier introduction of the notion see [6, 7]), which parametrizes new flavour effects by the same two Yukawa coupling matrices as they appear in the SM. Up to now all data in flavour physics, in particular from rare kaon and  $B$ -meson decays, indicate that new-physics contributions to flavour transitions are small. Models with new physics at the

TeV scale are therefore favorably formulated within an MFV scenario. On the other hand, MFV scenarios will shorten the lever arm for flavour physics experiments to discover and measure new physics in flavour transitions, since — except for the top quark — all these transitions involve small mixing angles and/or Yukawa couplings.

The case for a super  $B$  factory and the flavour-physics program at the LHC lies in the hope that nature may be at some not too high scale not minimal flavour violating. Again we do not have a compelling theory for such a scenario, but we may as well try to parameterize it. In the present paper, we discuss a possible parameterization in terms of additional spurion fields, which break the flavour symmetry in a different way as the two spurions associated with the Yukawa matrices present already in the SM. We will concentrate on the quark-flavour sector. Similar considerations could also be performed for lepton-flavour transitions, but will not be discussed in this paper. We will also stick to a simple scenario with one Higgs doublet, but should keep in mind that some flavour transitions can be enhanced by large  $\tan \beta = \langle H_1 \rangle / \langle H_2 \rangle$  in 2-Higgs models. See [5] and [8] for discussions within MFV.

The paper is organized as follows. In the next section, we briefly review the flavour structure following from the quark Yukawa couplings to the Higgs field. In section 3 we summarize the flavour coefficients for quark transitions within MFV. Section 4 represents the main part of our paper, where we give a possible — model independent — definition of next-to-minimal flavour violating scenarios. For this purpose we introduce additional spurion fields with different transformation properties under the flavour group and a particular hierarchy with respect to the Wolfenstein parameter  $\lambda$ . Two illustrative examples, where a new spurion — coupling exclusively to right-handed quarks — appears, are worked out in some more detail. We conclude in section 5.

## 2. Quark-flavours in the standard model

For quarks, the maximal flavour group which commutes with the gauge group of the SM is

$$F = \text{SU}(3)_{Q_L} \times \text{SU}(3)_{U_R} \times \text{SU}(3)_{D_R} \tag{2.1}$$

where  $Q_L$  denotes the weak doublets of left-handed quarks transforming as  $(3, 1, 1)$ ,  $U_R$  are the weak singlets of right-handed up-type quarks, transforming as  $(1, 3, 1)$ , and  $D_R$  are the weak singlets of right-handed down-type quarks, transforming as  $(1, 1, 3)$ . The Higgs and the gauge fields of the SM transform as singlets under all factors of the flavour group (2.1).

The Yukawa couplings of the SM break the flavour symmetry (2.1). This breaking can be described in terms of two spurion fields  $Y_U$  and  $Y_D$ , where  $Y_U$  is assumed to transform as  $(3, \bar{3}, 1)$  and  $Y_D$  as  $(3, 1, \bar{3})$ . The formally invariant terms with a single insertion of the spurions can be written as

$$-\mathcal{L}_{\text{yuk}} = \bar{Q}'_L H Y_D D'_R + \bar{Q}'_L H Y_U U'_R + \text{h.c.} . \tag{2.2}$$

with the quark fields in the electro-weak basis written as

$$D'_R = \begin{pmatrix} 0 \\ d'_R \end{pmatrix}, \quad U'_R = \begin{pmatrix} u'_R \\ 0 \end{pmatrix}, \tag{2.3}$$

and the Higgs field introduced as a  $2 \times 2$  matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_0 + i\chi_0 & \sqrt{2}\phi_+ \\ \sqrt{2}\phi_- & \phi_0 - i\chi \end{pmatrix}. \quad (2.4)$$

The VEV of the Higgs field is chosen to be  $\langle \phi_0 \rangle = v \neq 0$  while the spurions  $Y_U$  and  $Y_D$  are “frozen” to the observed values of the Yukawa couplings. This leads to a mass term contained in (2.2) and the mass eigenstates are obtained by diagonalizing the resulting mass matrices. This diagonalization procedure may be expressed by bi-unitary transformations from the group  $F$  given in (2.1),<sup>1</sup>

$$V_{u_L}^\dagger Y_U V_{u_R} = \sqrt{2} m_U^{\text{diag}}/v \equiv \hat{m}_U, \quad (2.5)$$

$$V_{d_L}^\dagger Y_D V_{d_R} = \sqrt{2} m_D^{\text{diag}}/v \equiv \hat{m}_D, \quad (2.6)$$

where  $V_{u_L}, V_{d_L} \in \text{SU}(3)_{Q_L}$ ,  $V_{u_R} \in \text{SU}(3)_{U_R}$  and  $V_{d_R} \in \text{SU}(3)_{D_R}$ . This defines the quark fields  $U, D$  in the mass eigenbasis

$$U'_L = V_{u_L} U_L, \quad U'_R = V_{u_R} U_R, \quad D'_L = V_{d_L} D_L, \quad D'_R = V_{d_R} D_R. \quad (2.7)$$

and the Yukawa interactions (2.2) are expressed as

$$-\mathcal{L}_{\text{yuk}} = \bar{Q}_L H \hat{m}_D D_R + \bar{Q}_L H \hat{m}_U U_R + \text{h.c.} \quad (2.8)$$

In the mass eigenbasis the gauge sector of the SM reads

$$\begin{aligned} \mathcal{L}_{\text{gauge}} &= \bar{Q}'_L i\not{D} Q'_L + \bar{U}'_R i\not{D} U'_R + \bar{D}'_R i\not{D} D'_R \\ &= (\bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + (R \rightarrow L)) + (\bar{u}_L V_{\text{CKM}} i\not{D} d_L + \text{h.c.}), \end{aligned} \quad (2.9)$$

where  $i\not{D}$  denotes the covariant derivative in the corresponding representation of  $\text{SU}(2)_L \otimes \text{U}(1)_Y$ . Here, the mismatch between  $V_{u_L}$  and  $V_{d_L}$  defines the CKM matrix,

$$V_{u_L}^\dagger V_{d_L} \equiv V_{\text{CKM}} \quad (2.10)$$

and induces charged flavour transitions between  $u_L$  and  $d_L$ .

Thus the only observable flavour-violating effects in the SM (as well as in all minimal flavour violating scenarios, to be discussed below) are the different quark masses and the relative rotation  $V_{\text{CKM}}$  between the two eigenbases defined by  $V_{u_L}$  and  $V_{d_L}$  in which  $Y_U$  and  $Y_D$  are diagonal. Notice that the rotations  $V_{u_R}$  and  $V_{d_R}$  are not observable in the SM.

## 2.1 The role of custodial $\text{SU}(2)$ in flavour physics

It is often argued that the solution of the flavour problem will happen at some very high scale, possibly even the Planck scale. However, there is a symmetry connecting the flavour mixing and some properties of the mass spectrum with the gauge structure. This “custodial” symmetry [9–11] is an exact symmetry of the Higgs sector but is broken by the

---

<sup>1</sup>We assume that the eigenvalues of the spurions are real and non-negative. This can always be achieved by an appropriate chiral rotation.

Yukawa couplings and the fact that only one generator of the right handed symmetry is gauged, yielding the weak hypercharge.

More precisely, the Higgs sector of the SM has a chiral  $SU(2)_L \times SU(2)_R$  symmetry, under which the quark and Higgs fields transform as

$$Q_L \sim (2, 1), \quad Q_R \sim (1, 2), \quad H \sim (2, 2).$$

It is broken down to the custodial  $SU(2)_{L+R}$  by the Higgs VEV  $\langle \phi_0 \rangle \neq 0$ . Under the remaining symmetry the three goldstone modes of the Higgs field in (2.4) transform as a triplet, while the left- and right-handed up- and down-quarks form a doublet each. In the SM, custodial  $SU(2)$  is explicitly broken by the gauge interactions and by the Yukawa couplings.

In case we enforce custodial  $SU(2)$  as an additional symmetry, i.e. we assume that the flavour group commutes with the chiral symmetry  $SU(2)_L \times SU(2)_R$ , the flavour group to be considered would reduce to

$$F_C = SU(3)_{Q_L} \times SU(3)_{U_R+D_R} \tag{2.11}$$

since the right handed up and down quarks form a doublet under  $SU(2)_{L+R}$ . For the Yukawa couplings of the left- and right-handed quarks this has the consequence that there is only a single spurion field  $Y_C$  transforming as  $(3, \bar{3})$  under (2.11). Furthermore, making use of the freedom implied by (2.11) we may diagonalize  $Y_C$  with the *same* transformation for up and down quarks. Therefore, the presence of an exact custodial  $SU(2)$  symmetry excludes the possibility of flavour mixing, and would imply a degeneracy between the up and the down quark in each family.

In many GUTs (for instance in  $SO(10)$ ) the right-handed up- and down-quarks of one family are assigned to the same multiplet of the gauge group. Thus custodial  $SU(2)$  is a subgroup of the GUT gauge group, and the possible flavour group collapses to (2.11), in which case the possible Yukawa couplings can be made diagonal and hence family mixing is absent.<sup>2</sup> This also means that the origin of flavour mixing should be at or below the GUT scale, and related to the scale where the breaking of custodial  $SU(2)$  occurs.

### 3. Minimal flavour violating new physics

The SM Lagrangian consists of all possible dimension-4 operators, and the effect of switching to mass eigenstates is the flavour mixing that appears in the charged currents. Using an effective field theory picture at the electroweak scale  $\mu \sim M_W$ , possible new physics effects (arising from some high scale  $\Lambda \gg M_W$ ) can be parameterized by higher-dimensional operators. Due to the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  gauge symmetry the lowest possible dimension for new operators involving quarks is six, and so a generic parameterization of new physics in this picture involves all possible dimension-six operators. While this concept is quite successful in the gauge sector, it involves too many parameters to be useful in the flavour sector.

---

<sup>2</sup>For a recent discussion of MFV within  $SU(5)$  GUT, see [12].

	$U_L$	$U_R$	$D_L$	$D_R$
$\bar{U}_L$	$V_{u_L}^\dagger Y_D Y_D^\dagger V_{u_L}$ $= V_{\text{CKM}}^\dagger \hat{m}_D^2 V_{\text{CKM}}^\dagger$	$V_{u_L}^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= V_{\text{CKM}}^\dagger \hat{m}_D^2 V_{\text{CKM}}^\dagger \hat{m}_U$	$V_{u_L}^\dagger V_{d_L}$ $= V_{\text{CKM}}$	$V_{u_L}^\dagger Y_D V_{d_R}$ $= V_{\text{CKM}}^\dagger \hat{m}_D V_{\text{CKM}}^\dagger$
$\bar{U}_R$	h.c.	$V_{u_R}^\dagger Y_U^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= \hat{m}_U V_{\text{CKM}}^\dagger \hat{m}_D^2 V_{\text{CKM}}^\dagger \hat{m}_U$	$V_{u_R}^\dagger Y_U^\dagger V_{d_L}$ $= \hat{m}_U V_{\text{CKM}}$	$V_{u_R}^\dagger Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_U V_{\text{CKM}}^\dagger \hat{m}_D V_{\text{CKM}}^\dagger$
$\bar{D}_L$	h.c.	h.c.	$V_{d_L}^\dagger Y_U Y_U^\dagger V_{d_L}$ $= V_{\text{CKM}}^\dagger \hat{m}_U^2 V_{\text{CKM}}$	$V_{d_L}^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= V_{\text{CKM}}^\dagger \hat{m}_U^2 V_{\text{CKM}}^\dagger \hat{m}_D$
$\bar{D}_R$	h.c.	h.c.	h.c.	$V_{d_R}^\dagger Y_D^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_D V_{\text{CKM}}^\dagger \hat{m}_U^2 V_{\text{CKM}}^\dagger \hat{m}_D$

**Table 1:** Minimal number of spurion insertions to generate flavour transitions between left- and right-handed up- and down quarks.

It has been widely advertised to use the assumption of minimal flavour violation in order to reduce the number of possible parameters. Qualitatively this means that also in the new physics sector only the quark masses and the CKM matrix are assumed to appear. A clear formulation of this concept has been given in [5], and we shall use this approach here as well.

Defining MFV in the sense of [5], we have to look at insertions of the spurions  $Y_U$  and  $Y_D$  between quark fields, which are consistent with the flavour group  $F$ . A complete list of the minimal number of insertions necessary to generate flavour transitions between left- and right-handed up- and down-quark fields is given in table 1. This includes the trivial case, i.e. no insertions at all for charged left-handed decays, leaving the CKM matrix as in the SM. On the other hand, for right-handed FCNC we need at least four spurion insertions. As a general rule, right-handed decays in MFV involve an additional quark-mass factor per right-handed field, and FCNC always involve at least two CKM elements. A special case of MFV is the weak effective Hamiltonian in the SM [13], where the generic flavour structures in table 1 are realized via box and penguin diagrams.

An important point to notice here is that the predictive power, following from the MFV assumption, is related to the fact that most of the flavour structures in table 1 involve at least one small CKM element and/or quark mass. Consequently, the higher the number of spurion insertions, the smaller the corresponding coefficient. An exception to this rule are charged  $t \rightarrow b$  transitions, where  $m_t/v$  and  $|V_{tb}|$  are of order one. Therefore, new contributions to right-handed  $t \rightarrow b$  transitions with  $\mathcal{O}(1)$  flavour coefficients can occur even in MFV.

From the possible MFV couplings for quark bilinears in table 1 one can easily construct the flavour couplings of all possible four-quark operators. The possible spin and colour structures are constrained as usual by Lorentz and gauge symmetry, but their specification is not relevant for the following discussion. In case of rare semileptonic decays  $q \rightarrow q' \ell^+ \ell^-$ , one would also have to take into account the lepton-flavour sector. For simplicity, we assume in the following, that the dominating effects come from new contributions to  $q \rightarrow q' Z(\gamma)$  with subsequent SM couplings of the gauge bosons to the lepton pair.

#### 4. Defining non-Minimal Flavour Violation (nMFV)

If MFV holds, the relative effects of new physics contributions to flavour transitions are as small as for flavour-diagonal processes. Actually, the present experimental results for rare kaon and  $B$ -meson decays show no evidence for inconsistencies with the SM, which can be taken as an indication that — if there is new physics around the TeV scale — it is close to MFV. On the other hand, if one allows for generic flavour transitions in higher-dimensional new-physics operators, one is forced to consider new-physics scales much larger than 1 TeV.

We may imagine an intermediate scenario, next-to-minimal flavour violation (nMFV), where the size of the suppression factors for specific flavour transitions is somewhere between generic and minimal flavour violation. In this section we are going to attempt a model-independent definition of nMFV scenarios, using again a spurion analysis for the effective theory at the electro-weak scale.<sup>3</sup>

Starting point are the quark bilinears in table 1 and their transformation under the flavour group  $F$ . There are 10 possible combinations of  $3_{L,U,D}$  and  $\bar{3}_{L,U,D}$ , namely the flavour-singlet  $(1, 1, 1)$  together with

$$\begin{array}{cccccc}
 (3, \bar{3}, 1) & (3, 1, \bar{3}) & (1, 3, \bar{3}) & (\bar{3}, 3, 1) & (\bar{3}, 1, 3) & (1, \bar{3}, 3) \\
 (\bar{8}, 1, 1) & (1, 8, 1) & (1, 1, 8) & & & 
 \end{array} \tag{4.1}$$

The SM Yukawa couplings only involve  $(3, \bar{3}, 1)$  and  $(3, 1, \bar{3})$  (and their conjugates), and therefore only the spurions  $Y_U$  and  $Y_D$  have to be considered. MFV is based on the assumption that  $Y_U$  and  $Y_D$  are sufficient to parameterize all relevant flavour transitions in new physics operators. A possibility to define nMFV is to allow for one (or more) additional elementary spurion fields from the following set,

$$\begin{array}{ll}
 Y_R \sim (1, 3, \bar{3}), & Y_R^\dagger \sim (1, \bar{3}, 3) \\
 Z_L = Z_L^\dagger \sim (8, 1, 1), & Z_U = Z_U^\dagger \sim (1, 8, 1), \quad Z_D = Z_D^\dagger \sim (1, 1, 8).
 \end{array} \tag{4.2}$$

In order to achieve predictive power, we have again to require that the elements of the new spurion fields show some hierarchy in terms of a small parameter  $\lambda'$  (similar, but not necessarily related to the Wolfenstein parameter  $\lambda$ ).

The new spurion fields and their combinations with the MFV spurions  $Y_U$  and  $Y_D$  give new (independent) possibilities to saturate the flavour structures in table 1. In some cases, one needs a *smaller* number of spurion insertions than in MFV, i.e. one generates potentially *larger* flavour coefficients. The possibility to combine nMFV and MFV spurion fields constrains the allowed power-counting for the nMFV spurions. For instance, the combination  $Y_U Y_R \sim (3, 1, \bar{3})$  transforms as  $Y_D$ , and therefore it can also appear at the corresponding place in the SM Yukawa term. In order to keep the SM power counting for CKM angles and quark masses, we thus have to require that

$$(Y_U Y_R)_{ij} \sim (\lambda^{(\prime)})^{n_{ij}} \leq (\lambda)^{m_{ij}} \sim (Y_D)_{ij} \quad \text{etc.} \tag{4.3}$$

---

<sup>3</sup>For an alternative approach, where nMFV is defined by new physics coupling dominantly to the third generation, see [14].

for all  $i, j$ , where  $n_{ij}$  and  $m_{ij}$  are some integer numbers specifying the power-counting in a given new-physics model. If these inequalities hold, we can always absorb the effects of nMFV spurions appearing in dim-4 operators into a redefinition of the MFV spurions  $Y_U$  and  $Y_D$ .

To illustrate our idea, we will, in the following, consider an example, where we include one nMFV spurion  $Y_R$ . In this case, we can express all possible flavour coefficients in terms of the quark masses, the CKM matrix and a new complex matrix<sup>4</sup>

$$R = V_{u_R}^\dagger Y_R V_{d_R}.$$

For instance,

$$V_{u_L}^\dagger Y_U Y_R V_{d_R} = \hat{m}_U V_{u_R}^\dagger Y_R V_{d_R} \equiv \hat{m}_U R, \tag{4.4}$$

$$V_{d_L}^\dagger Y_U Y_R V_{d_R} = V_{\text{CKM}}^\dagger \hat{m}_U V_{u_R}^\dagger Y_R V_{d_R} \equiv \hat{V}_{\text{CKM}}^\dagger \hat{m}_U R, \tag{4.5}$$

$$V_{d_L}^\dagger Y_D Y_R^\dagger V_{u_R} = \hat{m}_D V_{d_R}^\dagger Y_R^\dagger V_{u_R} \equiv \hat{m}_D R^\dagger, \tag{4.6}$$

$$V_{u_L}^\dagger Y_D Y_R^\dagger V_{u_R} = V_{\text{CKM}}^\dagger \hat{m}_D V_{d_R}^\dagger Y_R^\dagger V_{u_R} \equiv V_{\text{CKM}}^\dagger \hat{m}_D R^\dagger, \quad \text{etc.} \tag{4.7}$$

In particular, since  $Y_R$  exclusively couples to right-handed quarks, it can induce potentially large effects in right-handed flavour transitions which are suppressed by small Yukawa couplings in the SM. Of course, the size of the effects depends crucially on the assumed power-counting for the matrix elements  $R_{ij}$ . In the next subsections, we specify two examples, where the power counting for  $R_{ij}$  is fixed either within a simple Froggatt-Nielsen model, or assumed to be democratic (i.e. independent of  $i$  and  $j$ ).

#### 4.1 Example 1: nMFV spurion $Y_R$ and Froggatt-Nielsen power-counting

To illustrate the possible quantitative effects of nMFV flavour structures we use a (minimal) Froggatt-Nielsen scenario (FN) [15] (see also [16]). In this scenario the flavour transitions are due to interactions with some scalar field which breaks a hypothetical U(1) symmetry at a high scale. Different quark multiplets (in the weak eigenbasis) are supposed to have different charges under that symmetry:

$$Q_L^i : c + b_i, \quad U_R^i : c - a_i^u, \quad D_R^i : c - a_i^d. \tag{4.8}$$

The hierarchy of the Yukawa couplings then follows as

$$\begin{aligned} (Y_U)_{ij} &\sim \lambda^{|b_i + a_j|}, & (j = u, c, t) \\ (Y_D)_{ij} &\sim \lambda^{|b_i + a_j|}, & (j = d, s, b) \end{aligned} \tag{4.9}$$

where  $\lambda$  is the ratio of the VEV of the new scalar field and the new-physics scale, and is to be identified with the Wolfenstein parameter.  $SU(2)_L$  invariance requires  $b_u = b_d \equiv b_{u,d}$ ,  $b_c = b_s \equiv b_{c,s}$ ,  $b_t = b_b \equiv b_{t,b}$ . One further assumes  $a_i > 0$  and  $b_i \geq 0$ , together with the

---

<sup>4</sup>In models with right-handed gauge bosons  $W'$ , the matrix  $R$  can be identified with the CKM matrix  $V'_{\text{CKM}}$  in the right-handed sector. In this case,  $R^\dagger = R^{-1}$  is unitary. In the general nMFV scenario  $RR^\dagger \neq 1$ .



$a_u$	$a_c$	$a_t$	$a_d$	$a_s$	$a_b$	$b_{u,d}$	$b_{c,s}$	$b_{t,b}$	$m_u$	$m_c$	$m_t$	$m_d$	$m_s$	$m_b$
5	2	0	4	3	2	3	2	0	$\lambda^8$	$\lambda^4$	$\lambda^0$	$\lambda^7$	$\lambda^5$	$\lambda^2$
3	1	0	3	2	2	3	2	0	$\lambda^6$	$\lambda^3$	$\lambda^0$	$\lambda^6$	$\lambda^4$	$\lambda^2$

**Table 2:** Two examples for FN charges and the related Wolfenstein power-counting for quark masses. For simplicity, we fixed  $b_{t,b} = 0$ .

ordering  $a_u \geq a_c \geq a_t$ ,  $a_d \geq a_s \geq a_b$ , and  $b_{u,d} \geq b_{c,s} \geq b_{t,b}$ . The eigenvalues of up- and down-quark mass matrices follow as

$$m_i \sim \lambda^{b_i+a_i}.$$

The CKM elements scale as

$$(V_{\text{CKM}})_{ij} \sim \lambda^{|b_i-b_j|}.$$

The Wolfenstein counting for the CKM matrix thus fixes the differences for the charges  $b_i$ ,

$$b_{u,d} - b_{c,s} = 1, \quad b_{c,s} - b_{t,b} = 2, \quad b_{u,d} - b_{t,b} = 3.$$

Notice that the Wolfenstein counting for the quark masses can independently be controlled by the parameters  $a_i$ . Two phenomenologically acceptable examples are listed in table 2, where we fixed the unobservable  $b_{t,b} = 0$  for simplicity.

If we introduce other spurions with elementary transformations under the flavour group, the power-counting is fixed by the FN charges, too. For  $Y_R$ , in particular, we obtain

$$(Y_R)_{ij} \sim \lambda^{|a_i-a_j|} \quad (i = u, c, t; j = d, s, b)$$

For the first (second) example in table 2, the power counting reads

$$Y_R \sim \begin{pmatrix} \lambda^{1(0)} & \lambda^{2(1)} & \lambda^{3(1)} \\ \lambda^{2(2)} & \lambda^{1(1)} & \lambda^{0(1)} \\ \lambda^{4(3)} & \lambda^{3(2)} & \lambda^{2(2)} \end{pmatrix}. \quad (4.10)$$

Indeed, triangle inequalities between the FN charges guarantee that combinations of  $Y_R$  with  $Y_U$  or  $Y_D$  do not lead to larger terms than those already present in the SM,

$$(Y_D Y_R^\dagger)_{ij} \sim \lambda^{|b_i+a_{j'}+|-a_{j'}+a_j|} \leq \lambda^{|b_i+a_j|} \sim (Y_U)_{ij} \quad (4.11)$$

$$(Y_U Y_R)_{ij} \sim \lambda^{|b_i+a_{j'}+|-a_{j'}+a_j|} \leq \lambda^{|b_i+a_j|} \sim (Y_D)_{ij} \quad (4.12)$$

On the other hand, the elements of  $Y_R$  can be larger than the corresponding flavour structures that one can build from  $Y_U$  and  $Y_D$  in MFV,

$$(Y_U^\dagger Y_D)_{ij} \sim \lambda^{|a_i+b_k+|-b_k-a_j|} \leq \lambda^{|a_i-a_j|} \sim (Y_R)_{ij} \quad (4.13)$$

Below, we will systematically study the effect of  $Y_R$  insertions with FN power counting for charged and neutral flavour transitions with different chiralities.

decay	SM	MFV	rel. factor	nMFV	rel. factor
$\bar{U}_L D_L$	$ V_{UD} $	$ V_{UD} $	1	-	-
$\bar{U}_L D_R$		$\hat{m}_D  V_{UD} $	$\lambda^{ a_D+b_D }$	$Y_U Y_R$	$\lambda^{ a_U+b_U + a_U-a_D - b_U-b_D }$
$\bar{U}_R D_L$		$\hat{m}_U  V_{UD} $	$\lambda^{a_U+b_U}$	$Y_R Y_D^\dagger$	$\lambda^{ a_D+b_D + a_U-a_D - b_U-b_D }$
$\bar{U}_R D_R$		$\hat{m}_U \hat{m}_D  V_{UD} $	$\lambda^{a_U+b_U+a_D+b_D}$	$Y_R$	$\lambda^{ a_U-a_D - b_U-b_D }$

**Table 3:** Charged currents in MFV and nMFV. The global suppression factor for new-physics contributions to dim-6 operators is  $v^2/\Lambda_{\text{NP}}^2$ . The quoted relative suppression (or enhancement) factors are to be understood with respect to the leading left-handed SM transitions. The Wolfenstein power-counting refers to the Froggatt-Nielsen scenario with two alternatives for the power-counting of quark masses in the first (second) row of table 2.

**Charged decays.** The Wolfenstein power-counting for charged flavour transitions with different chiralities for MFV and nMFV (with FN power-counting for the spurion  $Y_R$ ) are summarized in table 3. Here we used that (in the mass basis) the entries of the matrix  $R$  have the same power counting as those of  $Y_R$ , since in FN the rotation matrices  $V_{u_R}$  and  $V_{d_R}$  are unity up to order  $\lambda$  effects. The interesting quantity is the suppression/enhancement factor, coming from the new possible flavour structures involving the spurion  $Y_R$ , relative to the leading left-handed tree-level transition in the SM. From the last column in table 3 we find

$$\bar{U}_L^i D_R^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{9(6)} & \lambda^{9(6)} & \lambda^{8(4)} \\ \lambda^{5(4)} & \lambda^{5(4)} & \lambda^{2(2)} \\ \lambda^{1(0)} & \lambda^{1(0)} & \lambda^2 \end{pmatrix}_{ij}, \quad (4.14)$$

$$\bar{U}_R^i D_L^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{8(6)} & \lambda^{6(4)} & \lambda^{2(0)} \\ \lambda^{8(7)} & \lambda^{6(5)} & \lambda^{0(1)} \\ \lambda^{8(6)} & \lambda^{6(4)} & \lambda^4 \end{pmatrix}_{ij}, \quad (4.15)$$

$$\bar{U}_R^i D_R^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{1(0)} & \lambda^{1(0)} & \lambda^{0(-2)} \\ \lambda^{1(1)} & \lambda^{1(1)} & \lambda^{-2(-1)} \\ \lambda^{1(0)} & \lambda^{1(0)} & \lambda^2 \end{pmatrix}_{ij}, \quad (4.16)$$

where the exponents refer to the power-counting for quark masses in the first (second) row of table 2. For purely left-handed transitions the spurion  $Y_R$  can only appear in combinations like  $Y_U Y_R Y_D^\dagger$  which are always smaller than  $V_{\text{CKM}}$  due to the triangle inequalities holding in FN.

**FCNCs involving down-quarks.** In table 4 we summarize the spurion combinations contributing to FCNCs with  $d$ -quarks in MFV and nMFV (with Wolfenstein power-counting for  $Y_R$  from FN). Again, for purely left-handed FCNC, insertions of  $Y_R$  cannot lead to larger effects than in MFV. For transitions with one or two right-handed down quarks, we find for the suppression/enhancement factors relative to the SM case,

$$\bar{D}_L^i D_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{1(0)} & \lambda^2 \\ \lambda^{1(0)} & - & \lambda^2 \\ \lambda^{1(0)} & \lambda^{1(0)} & - \end{pmatrix}_{ij}, \quad (4.17)$$

decay	SM + MFV		nMFV	rel. factor
$\bar{D}_L D'_L$	$Y_U Y_U^\dagger$	$\hat{m}_t^2  V_{tD'} V_{tD}^* $	-	-
$\bar{D}_L D'_R$	$Y_U Y_U^\dagger Y_D$	$\hat{m}'_D \hat{m}_t^2  V_{tD'} V_{tD}^* $	$Y_U Y_R$	$\lambda^{ a'_D  -  b'_D }$
$\bar{D}_R D'_R$	$Y_D^\dagger Y_U Y_U^\dagger Y_D$	$\hat{m}_D \hat{m}'_D \hat{m}_t^2  V_{tD'} V_{tD}^* $	$Y_R^\dagger Y_R$	$\max_u \left[ \lambda^{ a_D - a_u  +  a'_D - a_u  -  b_D  -  b'_D } \right]$

**Table 4:** FCNCs with down quarks in MFV and nMFV. The global suppression factor for new-physics contributions to dim-6 operators is  $16\pi^2 v^2 / \Lambda_{\text{NP}}^2$ . The quoted relative suppression (or enhancement) factors are to be understood with respect to the leading left-handed (loop-induced) SM transitions. The Wolfenstein power-counting refers to the Froggatt-Nielsen scenario with two alternatives for the power-counting of quark masses in the first (second) row of table 2.

decay	SM + MFV		nMFV	rel. factor
$\bar{U}_L U'_L$	$Y_D Y_D^\dagger$	$\hat{m}_b^2  V_{Ub} V_{U'b}^* $	-	-
$\bar{U}_L U'_R$	$Y_D Y_D^\dagger Y_U$	$\hat{m}'_U \hat{m}_b^2  V_{Ub} V_{U'b}^* $	$Y_D Y_R^\dagger$	$\lambda^{ a_b - a'_U  - a_b - b'_U}$
$\bar{U}_R U'_R$	$Y_U^\dagger Y_D Y_D^\dagger Y_U$	$\hat{m}_U \hat{m}'_U \hat{m}_b^2  V_{Ub} V_{U'b}^* $	$Y_R Y_R^\dagger$	$\max_d \left[ \lambda^{ a_U - a_d  +  a'_U - a_d  - b_U - b'_U - 2a_b} \right]$

**Table 5:** FCNCs with up quarks in MFV and nMFV. The global suppression factor for new-physics contributions to dim-6 operators is  $16\pi^2 v^2 / \Lambda_{\text{NP}}^2$ . The quoted relative suppression (or enhancement) factors are to be understood with respect to the leading left-handed (loop-induced) SM transitions. The Wolfenstein power-counting refers to the Froggatt-Nielsen scenario with two alternatives for the power-counting of quark masses in the first (second) row of table 2.

$$\bar{D}_R^i D_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{-2(-4)} & \lambda^{-1(-2)} \\ \lambda^{-2(-4)} & - & \lambda^{-1(0)} \\ \lambda^{-1(-2)} & \lambda^{-1(0)} & - \end{pmatrix}_{ij} . \quad (4.18)$$

To derive the second column in table 4, we have used that

$$V_{dL}^\dagger Y_U Y_R V_{dR} = V_{\text{CKM}}^\dagger \hat{m}_U R ,$$

and  $b_t = a_t = 0$ .

**FCNCs involving up-quarks.** In table 5 we summarize the spurion combinations contributing to FCNCs with  $u$ -quarks in MFV and nMFV (with Wolfenstein power-counting for  $Y_R$  from FN). Again, for purely left-handed FCNC, insertions of  $Y_R$  cannot lead to larger effects than in MFV. For transitions with one or two right-handed up quarks, we find for the suppression/enhancement factors relative to the SM case,

$$\bar{U}_L^i U_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{-4(-3)} & \lambda^0 \\ \lambda^{-2(-4)} & - & \lambda^0 \\ \lambda^{-2(-4)} & \lambda^{-4(-3)} & - \end{pmatrix}_{ij} , \quad (4.19)$$

$$\bar{U}_R^i U_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{-6(-7)} & \lambda^{-2(-4)} \\ \lambda^{-6(-7)} & - & \lambda^{-4(-3)} \\ \lambda^{-2(-4)} & \lambda^{-4(-3)} & - \end{pmatrix}_{ij} . \quad (4.20)$$

## 4.2 Example 2: nMFV spurion $Y_R$ with democratic power-counting

The simple FN scenario in the previous section clearly leads to rather large effects in certain flavour transitions and therefore should not be considered as phenomenologically favorable. An alternative and complementary approach would be to assume a *democratic* power counting for the new spurion fields (in the mass eigenbasis). Sticking again to the scenario with one nMFV spurion  $Y_R$ , we consider the power-counting

$$R_{ij} \sim \lambda^{4(3)} \quad (4.21)$$

which corresponds to the smallest entry in (4.10), where we consider again the Wolfenstein-scaling for quarks as in table 2. The relative suppression/enhancement factors with respect to the leading SM contributions in this case are as follows.

**Charged decays.**

$$\bar{U}_L^i D_R^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{12(9)} & \lambda^{11(8)} & \lambda^{9(6)} \\ \lambda^{7(5)} & \lambda^{8(6)} & \lambda^{6(4)} \\ \lambda^{1(0)} & \lambda^{2(1)} & \lambda^{4(3)} \end{pmatrix}_{ij}, \quad (4.22)$$

$$\bar{U}_R^i D_L^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{11(9)} & \lambda^{8(6)} & \lambda^{3(2)} \\ \lambda^{10(8)} & \lambda^{9(7)} & \lambda^{4(3)} \\ \lambda^{8(6)} & \lambda^{7(5)} & \lambda^{6(5)} \end{pmatrix}_{ij}, \quad (4.23)$$

$$\bar{U}_R^i D_R^j : \frac{v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} \lambda^{4(3)} & \lambda^{3(2)} & \lambda^{1(0)} \\ \lambda^{3(2)} & \lambda^{4(3)} & \lambda^{2(1)} \\ \lambda^{1(0)} & \lambda^{2(1)} & \lambda^{4(3)} \end{pmatrix}_{ij}, \quad (4.24)$$

**FCNCs involving down-quarks.**

$$\bar{D}_L^i D_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{2(1)} & \lambda^{4(3)} \\ \lambda^{1(0)} & - & \lambda^{4(3)} \\ \lambda^{1(0)} & \lambda^{2(1)} & - \end{pmatrix}_{ij}, \quad (4.25)$$

$$\bar{D}_R^i D_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{3(1)} & \lambda^{5(3)} \\ \lambda^{3(1)} & - & \lambda^{6(4)} \\ \lambda^{5(3)} & \lambda^{6(4)} & - \end{pmatrix}_{ij}. \quad (4.26)$$

**FCNCs involving up-quarks.**

$$\bar{U}_L^i U_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{0(-1)} & \lambda^{2(1)} \\ \lambda^{-1(-2)} & - & \lambda^{2(1)} \\ \lambda^{-1(-2)} & \lambda^{0(-1)} & - \end{pmatrix}_{ij}, \quad (4.27)$$

$$\bar{U}_R^i U_R^j : \frac{16\pi^2 v^2}{\Lambda_{\text{NP}}^2} \begin{pmatrix} - & \lambda^{-1(-3)} & \lambda^{1(-1)} \\ \lambda^{-1(-3)} & - & \lambda^{2(0)} \\ \lambda^{1(-1)} & \lambda^{2(0)} & - \end{pmatrix}_{ij}. \quad (4.28)$$

### 4.3 Phenomenological implications

As already stated in the introduction, the concept of MFV provides a natural explanation for the present success of the SM in reproducing the flavour observables in the CKM analysis, despite the possible existence of new physics at or slightly below the TeV scale. Within MFV the phenomenological determination of quantities like  $|V_{ub}|$  from  $b \rightarrow u\ell\nu$ ,  $|V_{td}/V_{ts}|$  from  $\Delta M_{B_d}/\Delta M_{B_s}$  or  $\Gamma[b \rightarrow d\gamma]/\Gamma[b \rightarrow s\gamma]$ , and  $\sin 2\beta$  from  $|a_{J/\psi K}^{\text{CP}}|$  is insensitive to new physics effects, even in 2-Higgs scenarios with large-tan  $\beta$  [5].

However, for the same reason, it will be difficult to really establish minimally flavour-violating new physics in flavour transitions. On the one hand, one has to identify small deviations from the SM. On the other hand, in order to exclude nMFV, one has to show that all flavour transitions are indeed driven by the CKM and mass factors as predicted by the analysis of [5]. In either case, one needs very good control on theoretical uncertainties.

As an example, let us consider FCNCs in the down-quark sector, such as  $b \rightarrow s$  and  $b \rightarrow d$  transitions. The dominating short-distance contribution within the standard model as well as possible new physics contributions in an MFV scenario are proportional to the combination  $|V_{ts}V_{tb}^*| m_t^2$  and  $|V_{td}V_{tb}^*| m_t^2$ , respectively, and hence the *relative* strength of the two processes will remain unchanged in MFV. Still, the analysis may be obscured by the problem of computing the relevant hadronic matrix elements, for instance the hadronic form factors for  $B \rightarrow K^*\gamma$  and  $B \rightarrow \rho\gamma$  decays [17–20]. Within the present uncertainties, the determination of  $|V_{td}/V_{ts}|$  from these decays is compatible with the global CKM fit [21], in particular with the complementary determination from  $\Delta M_{B_d}/\Delta M_{B_s}$ . From this we may conclude that new physics effects in these observables are either absent or suppressed via MFV. Improving the experimental and theoretical errors in both observables in the future might reveal a mismatch between the independent determinations of  $|V_{td}/V_{ts}|$  which would point towards non-minimal flavour violation.

In the following paragraphs we consider a few more examples, where we expect sizeable phenomenological implications within our particular ansatz for nMFV.

#### 4.3.1 nMFV: right handed spurion $Y_R$

By construction, the inclusion of an independent spurion  $Y_R$  enhances the possible new-physics effects for right-handed transitions, in particular it directly induces right-handed charged currents. Therefore, it should be worth looking into right-handed contributions to *charged*  $b \rightarrow u$  and  $b \rightarrow c$  decays, which may be significant despite the fact that the (left-handed) SM decay is not loop suppressed. In particular, the semileptonic  $b \rightarrow u$  and  $b \rightarrow c$  decays will be affected. The standard methods to extract the SM value for  $|V_{ub}/V_{cb}|$  from exclusive and inclusive decay modes might still be applicable in MFV scenarios, but in an nMFV scenario involving the right-handed spurion  $Y_R$  sizeable pollutions from right-handed quarks are expected to alter the result.

This may show up as an inconsistency like the presently observed 1- $\sigma$  tension between  $|V_{ub}/V_{cb}|$  and the  $\sin 2\beta$  value from  $B \rightarrow J/\psi K_s$  within the global fit of the CKM triangle [21]. It should also be stressed that nMFV contributions in exclusive and inclusive analyses will be rather different, and hence also the present tension between the exclusive

and inclusive value for  $|V_{ub}|$  could be attributed to such an effect. A strategy to directly test the left-handedness of  $b \rightarrow c$  transitions from a moment analysis of inclusive spectra has recently been discussed in [22].

Other effects of  $Y_R$  could show up in channels, in which the relative enhancement is pronounced by the fact that the SM contribution is strongly suppressed by the GIM mechanism. A well-known example is  $D^0$ - $\bar{D}^0$  mixing, which is predicted to proceed very slowly in the SM (see the reviews [23, 24] and references therein). The phenomenological analysis of  $D^0 - \bar{D}^0$  mixing is complicated by the presence of various contributions from different short- and long-distance scales to the off-diagonal term in the mass matrix

$$2m_D \left( M - \frac{i}{2}\Gamma \right)_{12} = \langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=-2} | D^0 \rangle + \sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=-1} | n \rangle \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=-1} | D^0 \rangle}{M_D - E_n + i\epsilon}. \quad (4.29)$$

In the SM the short-distance contributions in  $\mathcal{H}_{\text{eff}}^{\Delta C=-2}$  are dominated by box diagrams with down-type quarks from the first and second family

$$\mathcal{H}_{\text{eff}}^{\Delta C=-2} \simeq \frac{G_F^2}{4\pi^2} |V_{cs}^* V_{cd}|^2 \frac{(m_s^2 - m_d^2)^2}{m_c^2} (\mathcal{O} + 2\mathcal{O}') \quad (4.30)$$

where  $\mathcal{O} = [\bar{u}\gamma_\mu(1 - \gamma_5)c]^2$  and  $\mathcal{O}' = [\bar{u}(1 + \gamma_5)c]^2$ . The unitarity of the CKM matrix (neglecting the small contribution from  $V_{ub}$ ) leads to a double-GIM suppression. The power-counting w.r.t. the Wolfenstein parameter (table 2) yields

$$|V_{cs}^* V_{cd}|^2 \frac{(\hat{m}_s^2 - \hat{m}_d^2)^2}{\hat{m}_c^2} \sim \lambda^{14(12)}. \quad (4.31)$$

The contribution from bottom quarks in the loop, proportional to

$$(V_{cb}^* V_{ub})^2 \hat{m}_b^2 \sim \lambda^{14},$$

is usually neglected. Notice that the light quarks in the box diagram are off-shell by an amount of order  $m_c^2$  only, which explains the factor  $1/m_c^2$  in (4.30) and implies that the effective interactions in (4.30) are *not* entirely due to short-distance effects *at the electroweak scale*.

In contrast, new heavy particles (e.g. squarks or non-standard scalars) could induce  $|\Delta C| = 2$  transitions at genuinely short-distance scales. In MFV the flavour coefficient cannot be larger than  $(V_{cb}^* V_{ub})^2 \hat{m}_b^4 \sim \lambda^{18}$ , and again we do not expect any sizeable effects. In nMFV with spurion  $Y_R$ , we may, for instance, consider the contribution from purely right-handed four-quark operators

$$\mathcal{H}_{\text{eff}}^{\Delta C=-2} \ni \frac{c_{RR}}{\Lambda_{\text{NP}}^2} \left( \sum_D R_{uD} R_{cD}^* \right)^2 [\bar{u}_R \gamma_\mu c_R]^2 \quad (4.32)$$

The power counting for the flavour coefficient yields  $(\sum_D R_{uD} R_{cD}^*)^2 = \lambda^{6(4)}$  in the FN scenario (4.10). In the more conservative democratic scenario (4.21), we obtain  $\lambda^{16(12)}$ , which is close to/the same as in (4.31). In this case, the NP effects might still compete

with the SM ones, if the overall coefficient  $c_{RR}$  in (4.32) is due to tree-level processes and not loop-suppressed as in the SM. Similarly, right-handed nMFV operators in  $\mathcal{H}_{\text{eff}}^{\Delta C=-1}$  may significantly change the long-distance contributions to  $D^0-\bar{D}^0$  mixing in (4.29).

The short-distance contributions to  $\Delta F = 2$  transitions involving down-type quarks, which are relevant for  $K^0-\bar{K}^0$  and  $B^0-\bar{B}^0$  mixing, are dominated by internal top-quark loops in the SM. The comparison of SM/MFV and purley right-handed nMFV contributions to the  $|\Delta S| = 2$  Hamiltonian reads

$$\begin{aligned}
 \text{SM: } & \hat{m}_t^2 (V_{ts}^* V_{td})^2 \sim \lambda^{10}, \\
 \text{FN (4.10): } & \left( \sum_U R_{Us}^* R_{Ud} \right)^2 \sim \lambda^{6(2)} \\
 \text{democratic (4.21): } & \left( \sum_U R_{Us}^* R_{Ud} \right)^2 \sim \lambda^{16(12)}
 \end{aligned} \tag{4.33}$$

Similarly, for  $|\Delta B| = 2$  and  $|\Delta S| = 0$  one has

$$\begin{aligned}
 \text{SM: } & \hat{m}_t^2 (V_{tb}^* V_{td})^2 \sim \lambda^6, \\
 \text{FN (4.10): } & \left( \sum_U R_{Ub}^* R_{Ud} \right)^2 \sim \lambda^{4(2)} \\
 \text{democratic (4.21): } & \left( \sum_U R_{Ub}^* R_{Ud} \right)^2 \sim \lambda^{16(12)}
 \end{aligned} \tag{4.34}$$

and for  $|\Delta B| = |\Delta S| = 2$

$$\begin{aligned}
 \text{SM: } & \hat{m}_t^2 (V_{tb}^* V_{ts})^2 \sim \lambda^4, \\
 \text{FN (4.10): } & \left( \sum_U R_{Ub}^* R_{Us} \right)^2 \sim \lambda^{2(4)} \\
 \text{democratic (4.21): } & \left( \sum_U R_{Ub}^* R_{Ud} \right)^2 \sim \lambda^{16(12)}
 \end{aligned} \tag{4.35}$$

Therefore, the relative effect of nMFV contributions involving right-handed quarks might be sizeable in  $K^0-\bar{K}^0$  mixing if the power-counting for the matrix  $R_{ij}$  is close to the (probably unrealistic) FN scenario. In all other cases the nMFV effects will in general be less dramatic than in  $D^0-\bar{D}^0$  mixing. On the other hand, the hadronic uncertainties, in particular in the case of  $B^0-\bar{B}^0$  mixing, are under somewhat better control.

### 4.3.2 nMFV: octet spurions

We have seen in the above example, that including the nMFV spurion  $Y_R$  we can generate all possible quark-bilinear flavour structures with at most *two* spurion insertions (in contrast to up to four in MFV scenarios). Clearly, allowing for the complete set of nMFV spurions, each flavour transition with a particular chirality structure has its own spurion. Actually, in the simple FN scenario discussed above, the  $Z_{L,U,D}$  spurions are allowed and their power-counting is again fixed by the  $a_i$  and  $b_i$  quantum numbers. As a result of the

triangle inequalities, the  $\bar{Q}_L Q_L$ ,  $\bar{U}_R U_R$  and  $\bar{D}_R D_R$  transitions can have even larger flavour coefficients than in the  $Y_R$  scenario discussed above.

In cases, in which only left-handed new physics interactions appear, the only possible new elementary spurion is  $Z_L$ . An example of such a case is the Littlest Higgs Model with T-Parity [25–27] whose flavour structure has been considered in some detail in [28, 29]. In these models the left-handed standard fermions couple to left-handed mirror fermions via heavy gauge bosons. The flavour structure of these couplings is described by two unitary matrices  $V_{Hu}$  and  $V_{Hd}$ , which can be written as

$$V_{Hu} = V_H^\dagger V_{u_L}, \quad V_{Hd} = V_H^\dagger V_{d_L}, \quad (4.36)$$

and satisfy the constraint  $V_{Hu}^\dagger V_{Hd} = V_{CKM}$ . At low scales, the nMFV effects in this model appear due to the mass splitting of the mirror fermions, such that we may define

$$V_H M_{\text{mf}} V_H^\dagger = \bar{M}_{\text{mf}} + V_H \Delta M_{\text{mf}} V_H^\dagger \equiv \bar{M}_{\text{mf}} (1 + Z_L). \quad (4.37)$$

where  $M_{\text{mf}}$  is the diagonal mass matrix of the mirror fermions,  $\bar{M}_{\text{mf}}$  is their average mass and  $\Delta M_{\text{mf}}$  their mass splitting. If the relative mass splitting  $\Delta M_{\text{mf}}/\bar{M}_{\text{mf}}$  and/or the off-diagonal matrix elements in  $V_H$  are sufficiently small, the littlest Higgs models satisfy our criteria for nMFV.

The minimal super-symmetric extension of the SM introduces new flavour structures through the soft SUSY-breaking sector. The tri-linear squark-Higgs couplings transform in the same way as the SM Yukawas. If one does not allow for generic flavour violation, they are naturally described in MFV,  $A_{ij}^U \propto (Y_U)_{ij}$  and  $A_{ij}^D \propto (Y_D)_{ij}$ . The squark mass terms transform as octet spurions,  $Z_{L,U,D}$ .

Certainly, without a compelling theory of flavour-breaking within a given new-physics model, it will be extremely difficult to disentangle the effects of the nMFV spurions  $Y_R$ ,  $Z_{L,U,D}$ . Nevertheless, we think that the possibility to classify different flavour-breaking effects beyond MFV alone, may be helpful for phenomenological studies which aim to constrain the flavour sector of physics beyond the SM.

## 5. Conclusions

In this paper we have proposed a model-independent scheme to classify new physics contributions to flavour transitions beyond the popular assumption of minimal flavour violation. In the effective-field-theory approach to MFV, all flavour transitions can be expressed in terms of fundamental spurion fields  $Y_U$  and  $Y_D$  which transform as  $(3, \bar{3}, 1)$  and  $(3, 1, \bar{3})$  under the flavour group  $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ . In the mass eigenbasis,  $Y_U$  and  $Y_D$  are given in terms of quark masses and CKM elements.

We define next-to-minimal flavour violation (nMFV) by allowing new spurion fields (4.2), satisfying a particular power-counting in Wolfenstein- $\lambda$  which is constrained by the inequalities (4.3). Depending on the considered nMFV spurion and the assumed power-counting, we can enhance certain flavour decay channels with respect to the SM/MFV. We have worked out the specific example of an nMFV spurion  $Y_R \sim (1, 3, \bar{3})$  which couples to



right-handed quarks. We have found that  $Y_R$  can lead to sizeable new-physics contributions in neutral  $D$ -meson and kaon decays, as well as in charged right-handed  $b \rightarrow u$  and  $b \rightarrow c$  transitions. Our classification scheme may be helpful as a starting point for studies of flavour violation beyond the SM in the era of the new collider experiments at the LHC and precision measurements at Super-B factories (see [30]).

## Acknowledgments

This work was supported by the German Research Foundation (DFG) under contract No. MA1187/10-1, and by the German Minister of Research (BMBF, contract No. 05HT6PSA).

## References

- [1] J.C. Pati and A. Salam, *Unified lepton-hadron symmetry and a gauge theory of the basic interactions*, *Phys. Rev. D* **8** (1973) 1240.
- [2] H. Georgi and S.L. Glashow, *Unity of all elementary particle forces*, *Phys. Rev. Lett.* **32** (1974) 438.
- [3] H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, *Ann. Phys. (NY)* **93** (1975) 193.
- [4] A. Masiero and L. Silvestrini, *Two lectures on FCNC and CP-violation in supersymmetry*, [hep-ph/9711401](#).
- [5] G. D’Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [[hep-ph/0207036](#)].
- [6] M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, *Next-to-leading QCD corrections to  $B \rightarrow X_s \gamma$  in supersymmetry*, *Nucl. Phys. B* **534** (1998) 3 [[hep-ph/9806308](#)].
- [7] A.J. Buras, P. Gambino, M. Gorbahn, S. Jäger and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, *Phys. Lett. B* **500** (2001) 161 [[hep-ph/0007085](#)].
- [8] V. Cirigliano, B. Grinstein, G. Isidori and M.B. Wise, *Minimal flavor violation in the lepton sector*, *Nucl. Phys. B* **728** (2005) 121 [[hep-ph/0507001](#)].
- [9] M.J.G. Veltman, *Limit on mass differences in the Weinberg model*, *Nucl. Phys. B* **123** (1977) 89.
- [10] P. Sikivie, L. Susskind, M.B. Voloshin and V.I. Zakharov, *Isospin breaking in technicolor models*, *Nucl. Phys. B* **173** (1980) 189.
- [11] M.S. Chanowitz, M.A. Furman and I. Hinchliffe, *Weak interactions of ultraheavy fermions*, *Phys. Lett. B* **78** (1978) 285.
- [12] B. Grinstein, V. Cirigliano, G. Isidori and M.B. Wise, *Grand unification and the principle of minimal flavor violation*, *Nucl. Phys. B* **763** (2007) 35 [[hep-ph/0608123](#)].
- [13] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)].
- [14] K. Agashe, M. Papucci, G. Perez and D. Pirjol, *Next to minimal flavor violation*, [hep-ph/0509117](#).

- [15] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, *Nucl. Phys. B* **147** (1979) 277.
- [16] M. Leurer, Y. Nir and N. Seiberg, *Mass matrix models*, *Nucl. Phys. B* **398** (1993) 319 [[hep-ph/9212278](#)]; *Mass matrix models: the sequel*, *Nucl. Phys. B* **420** (1994) 468 [[hep-ph/9310320](#)].
- [17] S.W. Bosch and G. Buchalla, *The radiative decays  $B \rightarrow V\gamma$  at next-to-leading order in QCD*, *Nucl. Phys. B* **621** (2002) 459 [[hep-ph/0106081](#)].
- [18] A. Ali and A.Y. Parkhomenko, *Branching ratios for  $B \rightarrow \rho\gamma$  decays in next-to-leading order in  $\alpha_s$  including hard spectator corrections*, *Eur. Phys. J. C* **23** (2002) 89 [[hep-ph/0105302](#)].
- [19] M. Beneke, T. Feldmann and D. Seidel, *Exclusive radiative and electroweak  $b \rightarrow d$  and  $b \rightarrow s$  penguin decays at NLO*, *Eur. Phys. J. C* **41** (2005) 173 [[hep-ph/0412400](#)].
- [20] P. Ball and R. Zwicky,  *$|V_{td}/V_{ts}|$  from  $B \rightarrow V\gamma$* , *JHEP* **04** (2006) 046 [[hep-ph/0603232](#)].
- [21] CKMFITTER GROUP collaboration, J. Charles et al., *CP-violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J. C* **41** (2005) 1 [[hep-ph/0406184](#)], updated results and plots available at: <http://ckmfitter.in2p3.fr>.
- [22] B.M. Dassingier, R. Feger and T. Mannel, *Testing the left-handedness of the  $b \rightarrow c$  transition*, [hep-ph/0701054](#).
- [23] G. Burdman and I. Shipsey,  *$D^0$ - $\bar{D}^0$  mixing and rare charm decays*, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 431 [[hep-ph/0310076](#)].
- [24] A.A. Petrov, *Charm mixing in the standard model and beyond*, *Int. J. Mod. Phys. A* **21** (2006) 5686 [[hep-ph/0611361](#)].
- [25] N. Arkani-Hamed, A.G. Cohen, E. Katz and A.E. Nelson, *The littlest Higgs*, *JHEP* **07** (2002) 034 [[hep-ph/0206021](#)].
- [26] H.-C. Cheng and I. Low, *Little hierarchy, little Higgses and a little symmetry*, *JHEP* **08** (2004) 061 [[hep-ph/0405243](#)].
- [27] I. Low, *T parity and the littlest Higgs*, *JHEP* **10** (2004) 067 [[hep-ph/0409025](#)].
- [28] J. Hubisz, S.J. Lee and G. Paz, *The flavor of a little Higgs with T-parity*, *JHEP* **06** (2006) 041 [[hep-ph/0512169](#)].
- [29] M. Blanke et al., *Particle antiparticle mixing,  $\epsilon_K$ ,  $\Delta\Gamma_q$ ,  $A_{SL}^q$ ,  $A_{CP}(B_d \rightarrow \psi K_S)$ ,  $A_{CP}(B_s \rightarrow \psi\phi)$  and  $B \rightarrow X_{s,d}\gamma$  in the littlest Higgs model with T-parity*, *JHEP* **12** (2006) 003 [[hep-ph/0605214](#)]; *Rare and CP-violating K and B decays in the littlest Higgs model with T-parity*, *JHEP* **01** (2007) 066 [[hep-ph/0610298](#)].
- [30] *Flavour in the era of the LHC*, workshop on the interplay of flavour and collider physics, see website <http://mlm.home.cern.ch/mlm/FlavLHC.html>.